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An Orderings based on random variables and fuzzy environments

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Abstract

In this paper new definitions on the ideas of stochastic ordering, hazard rate ordering and the stochastically more factor ordering of fuzzy random variables are introduced. Different properties of these orderings are laid out.

Keywords: Fuzzy Random Variables, Stochastic Ordering, Hazard Rate Ordering

Introduction

The idea of fuzzy random variable, that expands the old style meaning of random variable, was presented by Féron in 1976. Later on, a few creators, and particularly Kwakernaak, Puri and Ralescu Kruse and Meyer Jewel and Kloeden proposed different variations. All the more as of late Krätschmer studied these definitions and proposed a brought together numerical proper methodology. In these papers, a fuzzy random variable is characterized as a capability that doles out a fuzzy subset to every conceivable result of a random examination. The various definitions in the writing differ on the quantifiability conditions forced to this planning, and in the properties of the result space, however every one of them mean to show circumstances that consolidate fluffiness and randomness. A fuzzy random variable taking qualities on the arrangement of fuzzy numbers, whose supports are in the genuine line then, at that point, sums up both the idea of random variable and the thought of span. A fascinating issue is then the accompanying: how to think about the overall greatness of two fuzzy random variables in concurrence with existing techniques for looking at the areas of random variables and existing strategies for contrasting spans ? Like for random sets, there is certainly not a remarkable instinct behind the different meanings of fuzzy random variables initially proposed in the writing. While Feron , Puri and Ralescu , Precious stone and Kloeden view a fuzzy random variable as the expansion of a random set to a random enrollment capability, Kwakernaak, and Kruse and Meyer consider that this participation capability models the uncertain impression of a badly known old style random variable. This disparity of perspectives, acquired from a similar one existing for random sets straightforwardly influences the decision of reasonable definitions for expansions of customary measurable methods to fuzzy random variables, as well with respect to fundamental thoughts like freedom, and molding.

The three understandings of fuzzy random variables

As brought up in late distributions, the thought of set, here a shut stretch for straightforwardness, might be utilized to demonstrate three kinds of data:

- Ontic circumstance: the exact depiction of a setvalued element. For example, the time span some activity has required to have been performed.
- Epistemic circumstance: The loose portrayal of a deterministic point-value ed amount. For example a period span addressing a specialist's information about a birth-date.
- Random epistemic circumstance: The loose portrayal of a random point-value ed amount. For example, a period stretch addressing the dispersion of temperatures in a day.

While a similar numerical substance is expected to address such circumstances, in the primary case the set addresses the assortment of its components (it is conjunctive), and tastes really ontic (the idea of the addressed item is set-hypothetical), while in the subsequent case, the set contains fundamentally unrelated qualities, one of which is the genuine one (the set is disjunctive) also, tastes epistemic. A similar applies, in the last circumstance, then again, actually now, we manage a disjunctive arrangement of (frequents) likelihood circulations, one of which being the genuine one. Similar qualifications can be made for fuzzy sets. A fuzzy stretch E˜ can represent an ontic substance, or an epistemic portrayal. For example, in the ontic view, it very well may be a steady time stretch, where the enrollment grade μE˜(r) addresses how much the activity performed during E˜ is locked in, with the possibility that this commitment continuously begins all along and stops smoothly, as opposed to unexpectedly. Running against the norm, in the epistemic view, the fuzzy stretch is a slow portrayal of the specialist's vulnerability about a badly realized point value,

point value, $\mu_{\tilde{E}}(r)$ being the degree of possibility $\pi_x(r)$ that $x = r$ [22]. Operationally, $\pi_x(r)$ can be viewed as the minimal selling price for a gamble that returns 1 euro if $x = r$, following the view of Walley [43]. Under a random epistemic view, $\tilde{E}$ represents the set of probability distributions $\mathcal{P}(\tilde{E}) = \{P : \forall A, P(A) \leq \Pi(A) = \sup_{r \in A} \mu_{\tilde{E}}(r)\}$, containing the actual distribution.

Comparing intervals

Let $A = [a_1, a_2]$ and $B = [b_1, b_2]$ be two intervals. Comparing the intervals A and B, we have four relations $>_i, i = 1, 2, 3, 4$, defined in [4] as follows:

1. $[a_1, a_2] \geq_1 [b_1, b_2] \Leftrightarrow a_1 \geq b_2$
2. $[a_1, a_2] \geq_2 [b_1, b_2] \Leftrightarrow a_1 \geq b_1$
3. $[a_1, a_2] \geq_3 [b_1, b_2] \Leftrightarrow a_2 \geq b_2$
4. $[a_1, a_2] \geq_4 [b_1, b_2] \Leftrightarrow a_2 \geq b_1$.

The relation $\geq_1$ is the strongest, $\geq_4$ is the weakest, $\geq_2$ and $\geq_3$ are of intermediary strength.

In the case of ontic intervals, these comparisons are akin to the proposals made by Allen [4] to compare time intervals.

In fact, if $[a_1, a_2]$ models an ill-known value x and $[b_1, b_2]$ and ill-known quantity y, $x \geq_1 y$ is a robust inequality since it holds whatever the values of x and y are; $x \geq_2 y$ expresses a pessimistic attitude (if the higher x and y, the better); $x \geq_3 y$ expresses an optimistic attitude; while $x \geq_4 y$ expresses an adventurous attitude, since it may well be that $y > x$ when their values are known. These relations are known in the literature. Denote $A = [a_1, a_2]$ and $B = [b_1, b_2]$ for short.

- $A >_1 B \Leftrightarrow \neg(B \geq_4 A)$. The strict relation $>_1$ is an interval order (Fishburn [25]). In the case of independence between random variables a and b, $P(a > b) = 1$ is generally equivalent to $Support(a) >_1 Support(b)$.
- The simultaneous use of $\geq_2$ and $\geq_3$: $A \succeq B$ if and only if $A \geq_2 B$ and $A \geq_3 B$. This is the canonical order induced by the lattice structure of intervals, equipped with the operations max and min extended to intervals: $A \succeq B \Leftrightarrow \max([a_1, a_2], [b_1, b_2]) = [a_1, a_2] \iff \min([a_1, a_2], [b_1, b_2]) = [b_1, b_2]$ (we call it *lattice interval* order).

It makes sense to use the latter ordering when comparing non-independent quantities x and y. For instance, if x and y depend on a parameter λ, so that $x = \lambda a_1 + (1 - \lambda)a_2$ and $y = \lambda b_1 + (1 - \lambda)b_2$, then $x > y, \forall\lambda$ implies $x \succeq y$, not $x >_1 y$.

Conclusion

This paper has proposed an efficient examination of the different approaches to expand stochastic orderings and stretch orderings to fuzzy random variables mutually. We desire to have persuaded the peruser that few perspectives can be visualized and that the right methodology will depend about the pretended by the fuzzy random variable in the given application, contingent on whether it portrays a random fuzzy article, or vulnerability about a random variable, or yet as random stretch where vulnerability relates to a badly seen restrictive likelihood measure.

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